4.6 Arithmetic Sequences

Essential Question

How can you use an arithmetic sequence to describe a pattern?

An **arithmetic sequence** is an ordered list of numbers in which the difference between each pair of consecutive **terms**, or numbers in the list, is the same.

EXPLORATION 1 Describing a Pattern

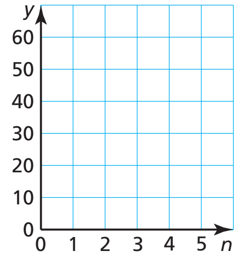
Work with a partner. Use the figures to complete the table. Plot the points given by your completed table. Describe the pattern of the *y*-values.

LOOKING FOR A PATTERN

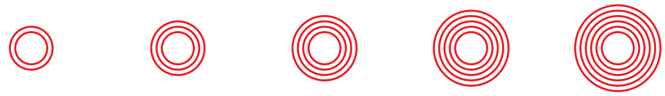
To be proficient in math, you need to look closely to discern patterns and structure.

a. $n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$

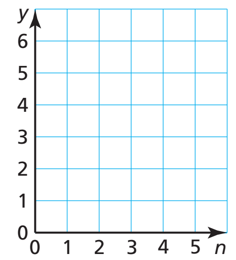
Number of stars, *n*	1	2	3	4	5
Number of sides, *y*					



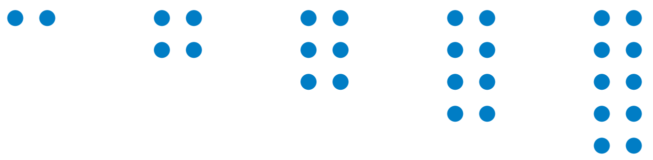
b. $n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$



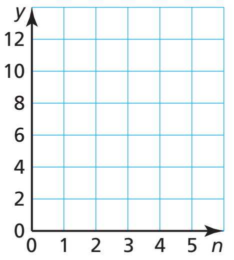
n	1	2	3	4	5
Number of circles, *y*					



c. $n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$



Number of rows, *n*	1	2	3	4	5
Number of dots, *y*					

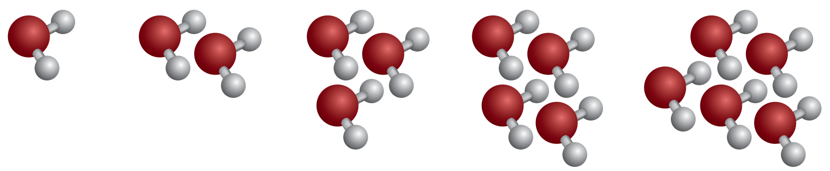


Communicate Your Answer

2. How can you use an arithmetic sequence to describe a pattern? Give an example from real life.

3. In chemistry, water is called H_2O because each molecule of water has two hydrogen atoms and one oxygen atom. Describe the pattern shown below. Use the pattern to determine the number of atoms in 23 molecules.

$n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$



4.6 Lesson

Core Vocabulary

sequence, *p. 210*
term, *p. 210*
arithmetic sequence, *p. 210*
common difference, *p. 210*

Previous
point-slope form
function notation

What You Will Learn

- Write the terms of arithmetic sequences.
- Graph arithmetic sequences.
- Write arithmetic sequences as functions.

Writing the Terms of Arithmetic Sequences

A **sequence** is an ordered list of numbers. Each number in a sequence is called a **term**. Each term a_n has a specific position n in the sequence.

$$5,\quad 10,\quad 15,\quad 20,\quad 25, \ldots, a_n, \ldots$$

1st position   3rd position   *n*th position

Core Concept

Arithmetic Sequence

In an **arithmetic sequence**, the difference between each pair of consecutive terms is the same. This difference is called the **common difference**. Each term is found by adding the common difference to the previous term.

$$5,\quad 10,\quad 15,\quad 20, \ldots$$

$+5 \quad +5 \quad +5$

Terms of an arithmetic sequence

common difference

READING

An ellipsis (. . .) is a series of dots that indicates an intentional omission of information. In mathematics, the . . . notation means "and so forth." The ellipsis indicates that there are more terms in the sequence that are not shown.

EXAMPLE 1 Extending an Arithmetic Sequence

Write the next three terms of the arithmetic sequence.

$-7, -14, -21, -28, \ldots$

SOLUTION

Use a table to organize the terms and find the pattern.

Position	1	2	3	4
Term	−7	−14	−21	−28

$+(-7) \quad +(-7) \quad +(-7)$

Each term is 7 less than the previous term. So, the common difference is −7.

Add −7 to a term to find the next term.

Position	1	2	3	4	5	6	7
Term	−7	−14	−21	−28	−35	−42	−49

$+(-7) \quad +(-7) \quad +(-7)$

▶ The next three terms are −35, −42, and −49.

Monitoring Progress

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Write the next three terms of the arithmetic sequence.

1. $-12, 0, 12, 24, \ldots$

2. $0.2, 0.6, 1, 1.4, \ldots$

3. $4, 3\frac{3}{4}, 3\frac{1}{2}, 3\frac{1}{4}, \ldots$

Graphing Arithmetic Sequences

To graph a sequence, let a term's position number n in the sequence be the x-value. The term a_n is the corresponding y-value. Plot the ordered pairs (n, a_n).

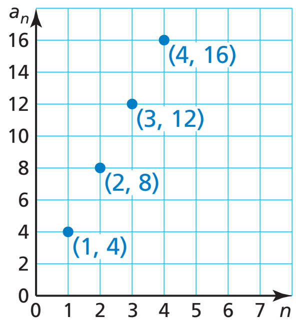
EXAMPLE 2 Graphing an Arithmetic Sequence

Graph the arithmetic sequence 4, 8, 12, 16, ... What do you notice?

SOLUTION

Make a table. Then plot the ordered pairs (n, a_n).

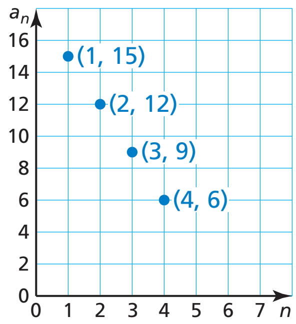
Position, n	Term, a_n
1	4
2	8
3	12
4	16



▶ The points lie on a line.

EXAMPLE 3 Identifying an Arithmetic Sequence from a Graph

Does the graph represent an arithmetic sequence? Explain.



SOLUTION

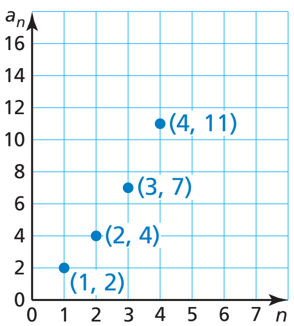
Make a table to organize the ordered pairs. Then determine whether there is a common difference.

Position, n	1	2	3	4
Term, a_n	15	12	9	6

$+(-3)$ $+(-3)$ $+(-3)$

Each term is 3 less than the previous term. So, the common difference is -3.

▶ Consecutive terms have a common difference of -3. So, the graph represents the arithmetic sequence 15, 12, 9, 6, ...



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Graph the arithmetic sequence. What do you notice?

4. 3, 6, 9, 12, . . . **5.** 4, 2, 0, -2, . . . **6.** 1, 0.8, 0.6, 0.4, . . .

7. Does the graph shown represent an arithmetic sequence? Explain.

Writing Arithmetic Sequences as Functions

Because consecutive terms of an arithmetic sequence have a common difference, the sequence has a constant rate of change. So, the points represented by any arithmetic sequence lie on a line. You can use the first term and the common difference to write a linear function that describes an arithmetic sequence. Let $a_1 = 4$ and $d = 3$.

Position, n	Term, a_n	Written using a_1 and d	Numbers
1	first term, a_1	a_1	4
2	second term, a_2	$a_1 + d$	$4 + 3 = 7$
3	third term, a_3	$a_1 + 2d$	$4 + 2(3) = 10$
4	fourth term, a_4	$a_1 + 3d$	$4 + 3(3) = 13$
⋮	⋮	⋮	⋮
n	nth term, a_n	$a_1 + (n - 1)d$	$4 + (n - 1)(3)$

ANOTHER WAY

An *arithmetic sequence* is a linear function whose domain is the set of positive integers. You can think of d as the slope and $(1, a_1)$ as a point on the graph of the function. An equation in point-slope form for the function is

$$a_n - a_1 = d(n - 1).$$

This equation can be rewritten as

$$a_n = a_1 + (n - 1)d.$$

Core Concept

Equation for an Arithmetic Sequence

Let a_n be the nth term of an arithmetic sequence with first term a_1 and common difference d. The nth term is given by

$$a_n = a_1 + (n - 1)d.$$

EXAMPLE 4 Finding the nth Term of an Arithmetic Sequence

Write an equation for the nth term of the arithmetic sequence 14, 11, 8, 5, ... Then find a_{50}.

STUDY TIP

Notice that the equation in Example 4 is of the form $y = mx + b$, where y is replaced by a_n and x is replaced by n.

SOLUTION

The first term is 14, and the common difference is -3.

$a_n = a_1 + (n - 1)d$ — Equation for an arithmetic sequence

$a_n = 14 + (n - 1)(-3)$ — Substitute 14 for a_1 and -3 for d.

$a_n = -3n + 17$ — Simplify.

Use the equation to find the 50th term.

$a_n = -3n + 17$ — Write the equation.

$a_{50} = -3(50) + 17$ — Substitute 50 for n.

$= -133$ — Simplify.

▶ The 50th term of the arithmetic sequence is -133.

Monitoring Progress

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Write an equation for the nth term of the arithmetic sequence. Then find a_{25}.

8. 4, 5, 6, 7, . . .

9. 8, 16, 24, 32, . . .

10. 1, 0, -1, -2, . . .

You can rewrite the equation for an arithmetic sequence with first term a_1 and common difference d in function notation by replacing a_n with $f(n)$.

$$f(n) = a_1 + (n - 1)d$$

The domain of the function is the set of positive integers.

EXAMPLE 5 Writing Real-Life Functions

Online bidding for a purse increases by $5 for each bid after the $60 initial bid.

Bid number	1	2	3	4
Bid amount	$60	$65	$70	$75

a. Write a function that represents the arithmetic sequence.

b. Graph the function.

c. The winning bid is $105. How many bids were there?

SOLUTION

a. The first term is 60, and the common difference is 5.

$f(n) = a_1 + (n - 1)d$ Function for an arithmetic sequence

$f(n) = 60 + (n - 1)5$ Substitute 60 for a_1 and 5 for d.

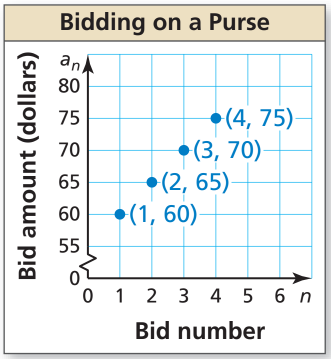
$f(n) = 5n + 55$ Simplify.

▶ The function $f(n) = 5n + 55$ represents the arithmetic sequence.

b. Make a table. Then plot the ordered pairs (n, a_n).

REMEMBER
The domain is the set of positive integers.

Bid number, n	Bid amount, a_n
1	60
2	65
3	70
4	75



c. Use the function to find the value of n for which $f(n) = 105$.

$f(n) = 5n + 55$ Write the function.

$105 = 5n + 55$ Substitute 105 for $f(n)$.

$10 = n$ Solve for n.

▶ There were 10 bids.

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Games	Total cost
1	$7
2	$9
3	$11
4	$13

11. A carnival charges $2 for each game after you pay a $5 entry fee.

a. Write a function that represents the arithmetic sequence.

b. Graph the function.

c. How many games can you play when you take $29 to the carnival?

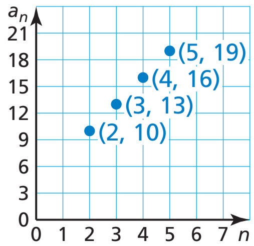
4.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. WRITING Describe the graph of an arithmetic sequence.

2. DIFFERENT WORDS, SAME QUESTION Consider the arithmetic sequence represented by the graph. Which is different? Find "both" answers.



Find the slope of the linear function.	Find the difference between consecutive terms of the arithmetic sequence.
Find the difference between the terms a_2 and a_4.	Find the common difference of the arithmetic sequence.

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, write the next three terms of the arithmetic sequence.

3. First term: 2
Common difference: 13

4. First term: 18
Common difference: –6

In Exercises 5–10, find the common difference of the arithmetic sequence.

5. 13, 18, 23, 28, . . .

6. 175, 150, 125, 100, . . .

7. $-16, -12, -8, -4, \ldots$

8. $4, 3\frac{2}{3}, 3\frac{1}{3}, 3, \ldots$

9. 6.5, 5, 3.5, 2, . . .

10. $-16, -7, 2, 11, \ldots$

In Exercises 11–16, write the next three terms of the arithmetic sequence. *(See Example 1.)*

11. 19, 22, 25, 28, . . .

12. 1, 12, 23, 34, . . .

13. 16, 21, 26, 31, . . .

14. $60, 30, 0, -30, \ldots$

15. 1.3, 1, 0.7, 0.4, . . .

16. $\frac{5}{6}, \frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \ldots$

In Exercises 17–22, graph the arithmetic sequence. *(See Example 2.)*

17. 4, 12, 20, 28, . . .

18. $-15, 0, 15, 30, \ldots$

19. –1, –3, –5, –7, . . .

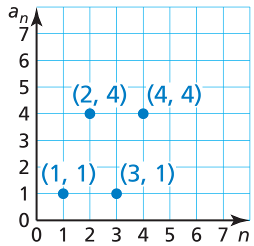
20. 2, 19, 36, 53, . . .

21. $0, 4\frac{1}{2}, 9, 13\frac{1}{2}, \ldots$

22. 6, 5.25, 4.5, 3.75, . . .

In Exercises 23–26, determine whether the graph represents an arithmetic sequence. Explain. *(See Example 3.)*

23.



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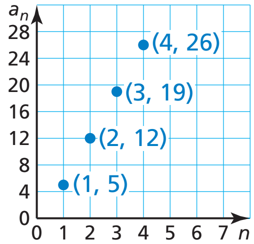

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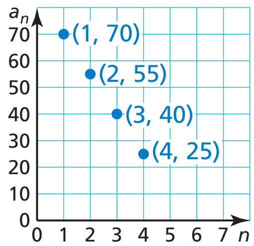

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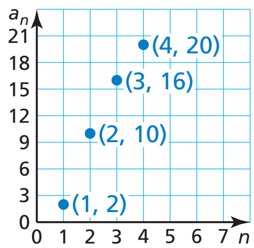


In Exercises 27–30, determine whether the sequence is arithmetic. If so, find the common difference.

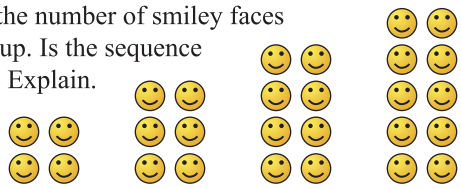
27. 13, 26, 39, 52, . . .

28. 5, 9, 14, 20, . . .

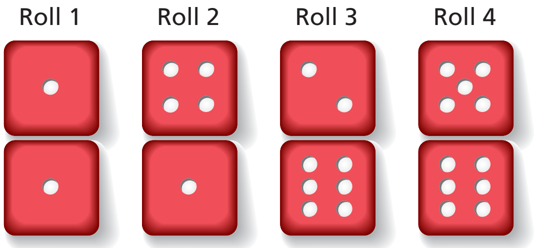
29. 48, 24, 12, 6, . . .

30. 87, 81, 75, 69, . . .

31. FINDING A PATTERN Write a sequence that represents the number of smiley faces in each group. Is the sequence arithmetic? Explain.



32. **FINDING A PATTERN** Write a sequence that represents the sum of the numbers in each roll. Is the sequence arithmetic? Explain.



In Exercises 33–38, write an equation for the nth term of the arithmetic sequence. Then find a_{10}. *(See Example 4.)*

33. $-5, -4, -3, -2, \ldots$

34. $-6, -9, -12, -15, \ldots$

35. $\frac{1}{2}, 1, 1\frac{1}{2}, 2, \ldots$

36. $100, 110, 120, 130, \ldots$

37. $10, 0, -10, -20, \ldots$

38. $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \ldots$

39. **ERROR ANALYSIS** Describe and correct the error in finding the common difference of the arithmetic sequence.

> ✗ $2, \quad 1, \quad 0, \quad -1, \ldots$
>
> $-1 \quad -1 \quad -1$
>
> The common difference is 1.

40. **ERROR ANALYSIS** Describe and correct the error in writing an equation for the nth term of the arithmetic sequence.

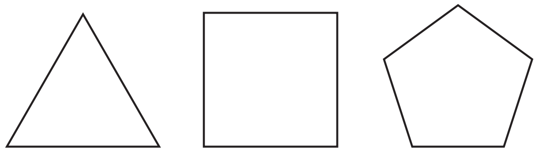
> ✗ $14, 22, 30, 38, \ldots$
>
> $a_n = a_1 + nd$
>
> $a_n = 14 + 8n$

41. **NUMBER SENSE** The first term of an arithmetic sequence is 3. The common difference of the sequence is 1.5 times the first term. Write the next three terms of the sequence. Then graph the sequence.

42. **NUMBER SENSE** The first row of a dominoes display has 10 dominoes. Each row after the first has two more dominoes than the row before it. Write the first five terms of the sequence that represents the number of dominoes in each row. Then graph the sequence.

REPEATED REASONING In Exercises 43 and 44, (a) draw the next three figures in the sequence and (b) describe the 20th figure in the sequence.

43.



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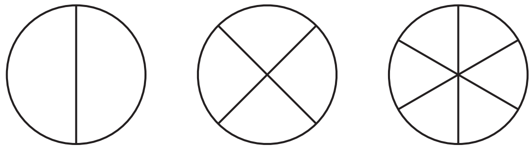


45. **MODELING WITH MATHEMATICS** The total number of babies born in a country each minute after midnight January 1st can be estimated by the sequence shown in the table. *(See Example 5.)*

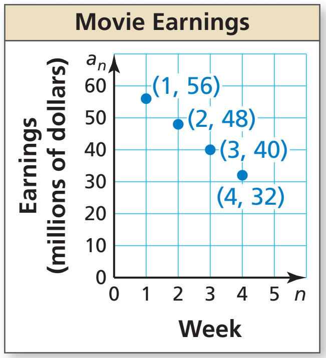
Minutes after midnight January 1st	1	2	3	4
Total babies born	5	10	15	20

a. Write a function that represents the arithmetic sequence.

b. Graph the function.

c. Estimate how many minutes after midnight January 1st it takes for 100 babies to be born.

46. **MODELING WITH MATHEMATICS** The amount of money a movie earns each week after its release can be approximated by the sequence shown in the graph.



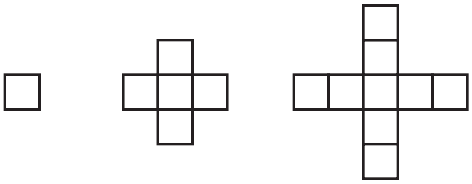
a. Write a function that represents the arithmetic sequence.

b. In what week does the movie earn $16 million?

c. How much money does the movie earn overall?

MATHEMATICAL CONNECTIONS In Exercises 47 and 48, each small square represents 1 square inch. Determine whether the areas of the figures form an arithmetic sequence. If so, write a function f that represents the arithmetic sequence and find $f(30)$.

47.



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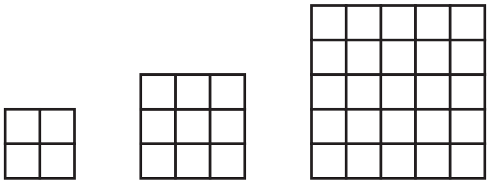


49. REASONING Is the domain of an arithmetic sequence discrete or continuous? Is the range of an arithmetic sequence discrete or continuous?

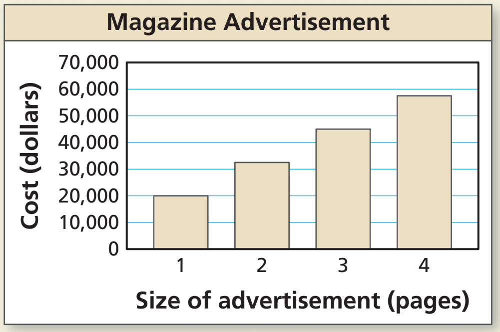
50. MAKING AN ARGUMENT Your friend says that the range of a function that represents an arithmetic sequence always contains only positive numbers or only negative numbers. Your friend claims this is true because the domain is the set of positive integers and the output values either constantly increase or constantly decrease. Is your friend correct? Explain.

51. OPEN-ENDED Write the first four terms of two different arithmetic sequences with a common difference of -3. Write an equation for the nth term of each sequence.

52. THOUGHT PROVOKING Describe an arithmetic sequence that models the numbers of people in a real-life situation.

53. REPEATED REASONING Firewood is stacked in a pile. The bottom row has 20 logs, and the top row has 14 logs. Each row has one more log than the row above it. How many logs are in the pile?

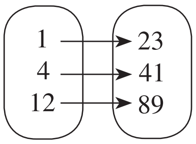
54. HOW DO YOU SEE IT? The bar graph shows the costs of advertising in a magazine.



a. Does the graph represent an arithmetic sequence? Explain.

b. Explain how you would estimate the cost of a six-page advertisement in the magazine.

55. REASONING Write a function f that represents the arithmetic sequence shown in the mapping diagram.



56. PROBLEM SOLVING A train stops at a station every 12 minutes starting at 6:00 A.M. You arrive at the station at 7:29 A.M. How long must you wait for the train?

57. ABSTRACT REASONING Let x be a constant. Determine whether each sequence is an arithmetic sequence. Explain.

a. $x + 6, 3x + 6, 5x + 6, 7x + 6, \ldots$

b. $x + 1, 3x + 1, 9x + 1, 27x + 1, \ldots$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the inequality. Graph the solution. *(Section 2.2)*

58. $x + 8 \geq -9$ **59.** $15 < b - 4$ **60.** $t - 21 < -12$ **61.** $7 + y \leq 3$

Graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range. *(Section 3.7)*

62. $h(x) = 3|x|$ **63.** $v(x) = |x - 5|$

64. $g(x) = |x| + 1$ **65.** $r(x) = -2|x|$